INTEREST RATES

This section would cover the following:

- Interest rates expressed in different time periods.
- ***Relationship between the rates of interest and discount*** over one effective period.
- Determine, when given a rate of interest under a specified payment frequency, the equivalent rate under an alternative payment frequency, including:
  - Annual effective rate of ***interest*** or ***discount***.
  - Rate of interest or discount payable p-thly ($p > 1$)
  - ***Force of interest***, which is the rate of interest applicable continuously over a period
- Accumulate a single investment at a constant rate of interest under the operation of ***simple interest*** and ***compound interest***.
- Determine the present value of a single investment at a constant rate of discount under the operation of ***simple discount*** and ***compound discount***.

Click here to see tab *Interest rates* in the shared Excel file online.

PRESENT VALUE AND FUTURE VALUES

Calculate the ***present value*** and ***accumulated value*** for a given stream of cashflows.
This can be simplified to get the present and accumulated values for a lumpsum.
These can be calculated by using direct functions or by using the first principles.
Understand how to calculate the present value for ***Perpetuities*** i.e., stream of cashflow for an infinite period of time.
Click here to see tab *Annuity PV and AV* in the shared Excel file online.

USING GOALSEEK

This is an important functionality in MS Excel useful for all practical calculations involving time value of money and ***equations of value***.
Click here to see tab *Goalseek* in the shared Excel file online.

LOAN SCHEDULE

A very common transaction involving compound interest is a ***loan*** that is repaid by regular instalments, at a fixed rate of interest, for a predetermined term.
Each repayment must pay first for ***interest due*** on the outstanding capital. The balance is then used to repay some of the ***capital outstanding***. Each payment, therefore, comprises both interest and capital repayment. It may be necessary to identify the separate elements of the payments – for example, if the tax treatment of interest and capital differs. Notice also that, where repayments are level, the interest component of the repayment instalments will decrease as capital is repaid, with the consequence that the capital payment will increase. Alternatively, the portion of capital repaid may remain constant in each period while the interest portion varies. This will result in ***decreasing repayments*** over time as interest is calculated on declining loan balance.
These ***calculations*** would often involve the use of Goalseek to calculate a repayment amount that would make the Loan outstanding at the end of the term equal to zero.
The loan may also be repaid through ***non-level payments***. If the repayment instalment is not sufficient to cover the interest portion of the loan, the ***loan outstanding increases*** as the unpaid interest is added to the loan.
In an ***interest-only loan***, each repayment instalments consists only of the interest portion of the loan and the principal borrowed may be repaid as a lumpsum at the end of the loan period.

To ensure that consumers can make informed judgements about the interest rates charged, lenders are required (in most circumstances) to give information about the effective rate of interest charged. In the UK this is in the form of the Annual Percentage Rate of charge, or ***APR***, which is defined as the effective annual rate of interest, rounded to the nearer 1/10th of 1%.

Loan calculations also involve finding the ***total interest paid*** or the total loan balance either using the ***prospective*** or the ***retrospective method***.

Click here to see tab *Loan schedule* in the shared Excel file online.

PROJECT APPRAISAL

Suppose an investor is considering the merits of an investment or business project. The investment or project will normally require an initial outlay and possibly other outlays in future, which will be followed by receipts, although in some cases the pattern of income and outgo is more complicated. The cashflows associated with the investment or business venture may be completely fixed (as in the case of a secure fixed-interest security maturing at a given date) or they may have to be estimated. The estimation of the cash inflows and outflows associated with a business project usually requires considerable experience and judgement. All the relevant factors (such as taxation and investment grants) and risks (such as construction delays) should be considered by the actuary, with assistance from experts in the relevant field.

For a scenario involving assessing the project, the idea would be to first plot the cashflows (negative or positive) based on time scale (example, cashflow at time 0, time 1, time 2, ... and so on) and then work out the following to decide whether investing in the project would be viable.

- ***Net Present Value*** (NPV): if NPV at target return is positive, it indicates a gain for the investor.
- ***Internal Rate of Return*** (IRR): if IRR is more than the target return, it indicates a gain for the investor.
- ***Discounted Payback Period*** (DPP): The sooner this is, the better it is for the investor to have the project as cashflow positive. One drawback with this measure is that, if there are negative cashflows in the future after the DPP point, this measure will not take that into account.

Click here to see tab *Project Appraisal* in the shared Excel file online.

PRICE OF PROPERTY

This is an application of theory of interest rates and equation of value where we will understand how to calculate the price of a property given the rental income, increase if any, sale price of the property and the required rate of return.

Click here to see tab *Price of Property* in the shared Excel file online.

FIXED-INTEREST SECURITY

Fixed interest securities are those that pay fixed interest payments throughout the term of the security. Example is a ***fixed interest bond*** with coupon payment of 3% per annum. Calculating the ***purchase price*** or ***redemption value*** of the bond. Considering ***taxation of income*** and ***capital gains*** to determine the price, or the ***effective net yield*** on the security.

Click here to see tab *Fixed-Interest Security* in the shared Excel file online.

INDEX-LINKED BONDS

If inflationary pressures in the economy are not kept under control, the purchasing power of a given sum of money diminishes with the passage of time, significantly so when the ***rate of inflation*** is high. For this reason, some investors are attracted by a security for which the actual cash amount of interest payments and of the final capital repayment are linked to an 'index' which reflects the effects of inflation.

Here, the initial negative cashflow is followed by a series of unknown positive cashflows and a single larger unknown positive cashflow, all on specified dates. However, it is known that the amounts of the future cashflows relate to the inflation index. Hence these cashflows are said to be known in ***'real' terms***.

In practice the operation of an ***index-linked security*** will be such that the cashflows do not relate to the inflation index at the time of payment, due to delays in calculating the index, called the 'time lag' which also needs to be considered when determining the cashflows.

Click here to see tab *Index-Linked Bond* in the shared Excel file online.

TERM STRUCTURE OF INTEREST RATE

Interest rate offered on investments does usually vary according to the term of the investment.
The yield on a unit ***zero coupon bond*** with term n years, yn , is called the ***n-year spot rate of interest***. The variation by term of interest rates is often referred to as the term structure of interest rates. The curve of ***spot rates*** {yt} is an example of a ***yield curve***.
The discrete time forward rate, $f(t, r)$, is the annual interest rate agreed at time 0 for an investment made at time $t > 0$ for a period of r years.
We will understand calculating the Discounted Mean Term, Volatility and Convexity which would determine vulnerability of cashflows to interest rate movements and hence cover ***Redington's 3 conditions for Immunization***.
The conditions for ***Redington's immunisation*** may be summarised as follows:

1. $VA(i_0) = VL(i_0)$ – that is, the value of the assets at the starting rate of interest is equal to the value of the liabilities.
2. The volatilities of the asset and liability cashflow series are equal, that is, $vA(i_0) = vL(i_0)$.
3. The convexity of the asset cashflow series is greater than the convexity of the liability cashflow series – that is, $cA(i_0) > cL(i_0)$.

Click here to see tab *Term Structure of Interest Rate* in the shared Excel file online.

CALCULATING LIFE TABLE FUNCTIONS

This section would cover calculating the following from a given ***Life Table*** (example used AM92):

- L_x and its select equivalents $l_{[x]+r}$
- d_x and its select equivalents $d_{[x]+r}$
- P_x
- μ_x which is the continuously compounded rate of decrement

This will form the base for all calculations involving contingencies.
Click here to see tab *Life Table* in the shared Excel file online.

GROSS FUTURE LOSS RANDOM VARIABLE

Consider the ***net random future loss*** (or just 'net loss') from a policy which is in force – where the net loss, L, is defined to be:

$$L = \text{present value of the future outgo—present value of the future income}$$

Now L is a random variable, since both terms are random variables which depend on the policyholder's future lifetime. (If premiums are not being paid, the second term is zero, the first term is a random variable, so L is still a random variable). When the outgo includes benefits and expenses, and the income is the gross premiums, then L is referred to as the ***gross future loss random variable***.
Premiums (and reserves) can be calculated which satisfy probabilities involving the gross future loss random variable.
Click here to see tab *Gross future loss random variab* in the shared Excel file online.

EXPECTED PRESENT VALUE AND BONUS CALCULATIONS

Understand how to calculate expected present values for life insurance product with given basis which can include mortality table, interest rate, expense and commission.
Different types of ***Reversionary Bonuses*** given by companies to policyholders on ***Conventional With-Profit contracts***:

- **Simple Bonus:** the rate of bonus each year is a percentage of the initial (basic) sum assured under the policy. The sum assured will increase linearly over the term of the policy.
- **Compound Bonus:** the rate of bonus each year is a percentage of the basic sum assured and the bonuses added in the past. The sum assured increases exponentially over the term of the policy.

- **Super Compound Bonus:** two compound bonus rates are declared each year. The first rate (usually the lower) is applied to the basic sum assured. The second rate is applied to the bonuses added to the policy in the past. The sum assured increases exponentially over the term of the policy. The sum assured, including bonuses, increases more slowly than under a compound allocation in the earlier years but faster in the later years.

The company may also give a ***Terminal Bonus*** at policy maturity or the event of claim.
Click here to see tab *EPV and Bonus* in the shared Excel file online.

WITH-PROFITS GROSS PREMIUM CALCULATION

The income to a life insurer comes from the payments made by policyholders, called the premiums. The outgo arises from benefits paid to policyholders and the insurer's expenses.
The ***gross premium*** is the premium required to meet all the costs under an insurance contract, and is the premium which the policyholder pays. When we talk of 'the premium' for a contract, we mean the gross premium. It is also sometimes referred to as the office premium.
Given a suitable set of assumptions, which we call the basis, we may use the equation of value to calculate the premium which a policyholder must pay in return for a given benefit. We may also calculate the amount of benefit payable by the insurer for a given premium.
The gross premium for a contract, given suitable mortality, interest and expense assumptions can be found from the equation of expected present value.
The ***equivalence principle*** states that $E[\text{gross future loss}] = 0$ which implies that:

$$E[\text{present value of benefits}] + E[\text{present value of expenses}] = E[\text{present values of premiums}]$$

so that in an expected present value context the premiums are equal (equivalent) in value to the expenses and the benefits. This relationship is usually called an equation of (gross expected present) value.
Click here to see tab *With-Profits Gross Premium* in the shared Excel file online.

PROSPECTIVE RESERVE

The ***prospective reserve*** for a life insurance contract which is in force (that is, has been written but has not yet expired through claim or reaching the end of the term) is defined to be, for a given basis:
The expected present value of the future outgo − The expected present value of the future income.
This is the ***prospective reserve*** because it looks forward to the future cashflows of the contract. The prospective reserve is important because if the company holds funds equal to the reserve, and the future experience follows the reserve basis, then, averaging over many policies, the combination of reserve and future income will be sufficient to pay the future liabilities. The reserve, therefore, gives the office a measure of the minimum funds it needs to hold at any point during the term of a contract. The process of calculating a reserve is called the ***valuation of the policy***.
Click here to see tab *Prospective Reserve_1* in the shared Excel file online.
Click here to see tab *Prospective Reserve_2* in the shared Excel file online.
Click here to see tab *Prospective Reserve_3* in the shared Excel file online.

DEATH STRAIN AT RISK & MORTALITY PROFIT

If experience does not follow the reserve basis, there will either be an excess of income over outgo (a profit, or surplus) or an excess of outgo over income (a loss or negative profit). Profits and losses may arise from any element of the reserve basis. Example - if the experienced mortality is heavier than that assumed in the basis, then there will be a ***profit or loss from mortality***, depending on the nature of the contract. Where benefits are paid out on death, such as a ***term assurance***, lighter mortality than assumed will give rise to a profit. Where benefits are paid out on survival such as an annuity, then lighter mortality will give rise to a loss.
The word strain is used loosely to mean a cost to the company. In words, the reasoning is that for each policy we must pay out at least $(t+1)V$ at the end of the year. In addition, if the policy becomes a claim during the year, with probability $q(x+t)$, then we must pay out an extra sum of $(S-(t+1)V)$ which is the ***DSAR***. Note that $q(x-t)$ is the probability of dying in the year t to $t+1$, and therefore $x+t$ is the age at the start of the year.

The expected amount of the death strain is called the ***expected death strain*** (EDS). This is the amount that the life insurance company expects to pay in addition to the year-end reserve for the policy. The probability of claiming in the policy year t to $t+1$ is $q(x+t)$ so that:

$$EDS = q(x+t) \times (S\text{–}(t-1)V)$$

The ***actual death strain*** (ADS) is simply the observed value at $t+1$ of the death strain random variable.
The ***mortality profit*** is defined as: Expected Death Strain - Actual Death Strain
The EDS is the amount the company expects to pay out, in addition to the year end reserve for a policy. The ADS is the amount it actually pays out, in addition to the year-end reserve. If it actually pays out less than it expected to pay, there will be a profit. If the actual strain is greater than the expected strain, there will be a loss.
Click here to see tab *Death Strain and Mort Profit* in the shared Excel file online.

RETROSPECTIVE ACCUMULATION

The basic idea is that we consider a group of lives, who are regarded as identical and stochastically independent as far as mortality is concerned. At age x, each life transacts an identical life insurance contract. Under these contracts, payments will be made (the direction of the payments is immaterial), depending on the experience of the members of the group. We imagine these payments being accumulated in a fund at rate of interest i.
After n years, we divide this fund equally among the surviving members of the group. (If the fund is negative, we imagine charging the survivors in equal shares). The ***retrospective accumulation*** is defined as the amount that each survivor would receive, as the group size tends towards infinity.
Click here to see tab *Retrospective Accumulation* in the shared Excel file online.

RETROSPECTIVE RESERVE

The ***retrospective reserve*** for a life insurance contract which is in force is defined to be, for a given basis: The accumulated value allowing for interest and survivorship of the premiums received to date − the accumulated value allowing for interest and survivorship of the benefits and expenses paid to date.
The retrospective reserve on a given basis tells us how much the premiums, less claims and expenses, have accumulated to, averaging over a large number of policies.
Gross premium retrospective reserves at policy duration t take account of expected expenses incurred between policy duration 0 and policy duration t, as well as the expected premiums and benefits paid during that period. All the expected present values are determined at policy duration, t.
Click here to see tab *Retrospective Reserve* in the shared Excel file online.

CALCULATING PROBABILITIES FOR TWO LIVES

This section would cover calculating various probabilities involving two lives.
This will form the base for all calculations involving ***contingencies***.
Click here to see tab *Life Table - 2 lives* in the shared Excel file online.
Click here to see tab *Prob involving 2 lives* in the shared Excel file online.

JOINT LIFE PREMIUM CALCULATION

This section will cover calculating premium for a ***joint life policy*** where benefit is paid on death of first life. This is similar to the one for single life, i.e., uses principle of equivalence but probabilities involving two lives are used.
Click here to see tab *Joint Life Premium Calculation* in the shared Excel file online.

JOINT LIFE RESERVES UNDER DIFFERENT SCENARIOS

This section covers calculating ***reserves for joint life*** scenarios where either one life is alive and both are alive. This is similar to the one for single life, i.e., (EPV of Outgo − EPV of Inflow) but probabilities involving two lives are used.
Click here to see tab *Joint Life Reserves* in the shared Excel file online.

MULTIPLE DECREMENTS

A multiple decrement model is a multiple state model which has:

- one active state; and

- one or more absorbing exit states.

This section will cover the calculation of dependent and independent probabilities in such a model using transition force and construct ***multiple decrement table***.
By 'dependent', we mean in the presence of all other risks of decrement in the population.
By 'independent', we mean when there is the only risk of decrement acting on the population so qx would be independent probability in a single decrement model.
Click here to see tab *Multiple Decrements* in the shared Excel file online.
Click here to see tab *Expected Cashflows without surr* in the shared Excel file online.

PROFIT TESTING FOR NON-UNIT LINKED CONTRACTS

The expected cashflows, both positive and negative, are used to construct a projected revenue account (per contract in force at the start of the period) for each time period. The balancing item in the projected revenue account is the profit emerging at the end of the time period.
In order to calculate the ***expected cashflows***, the following information is needed:

- Premiums received and their times of payment
- Expected expenses (from the basis) and their times of payment
- Contingent benefits payable under the contract, e.g. death benefit, annuity payment, survival benefit for endowment, difference between guaranteed sum assured and value of unit fund for unit-linked endowment
- Other benefits payable under the contract, e.g. surrender values
- Other expected cash payments, e.g. taxes
- Other expected cash receipts, e.g. management charges levied on a unit fund
- The ***reserves required*** for a contract, usually at the beginning and end of the time period, calculated using the ***valuation basis*** together with the different probabilities of the various events leading to the payment of particular cash amounts. Any balance on the expected revenue account during the time period will be invested, and an assumption about the rate of investment return is needed. This allows the expected investment income during the period to be calculated and credited at the end of the period.

The vector of balancing items in the projected revenue accounts for each policy year is called the ***profit vector***, $(PRO)_t$, $t = 1, 2, 3, \ldots$. The profit vector gives the expected profit at the end of each policy year per policy in force at the beginning of that policy year.
The vector of expected profits per policy issued is called the ***profit signature***, $(PS)_t$, $t = 1, 2, 3, \ldots$. This is obtained by multiplying the profit vector by the probability of a policy remaining in force from policy duration 0 to policy duration t. The vector representing the profit signature can be displayed graphically to illustrate the way in which profits are expected to emerge over the lifetime of the policy.
Summary measures usually involve determining the present values of the expected cashflows. In some cases, this requires an assumption about the ***discount rate***. This rate is chosen to equal the cost of capital plus a risk premium. The ***cost of capital*** is the rate at which funds can be borrowed, or the rate of return such funds would earn if invested elsewhere, (i.e. the 'opportunity cost'). The risk premium reflects the risks and uncertainties surrounding the cashflows to and from the policy, and is called the risk discount rate, id.
Net Present Value (NPV): This is the present value of the profit signature determined using the risk discount rate. The NPV can be interpreted as the EPV of the future profits from the policy, for a single policy as at the start date of the contract.
Profit Margin: This is the NPV expressed as a percentage of the EPV of the premium income
Internal Rate of Return (IRR): Discount rate that would make the NPV of the contract equal to zero. It does exist where the profit signature has a ***single financing phase***, i.e. where the first profit flow is negative and the profit flows in all subsequent years are positive.
Click here to see tab *Profit Testing_1* in the shared Excel file online.
Click here to see tab *Expected Cashflows with surr* in the shared Excel file online.
Click here to see tab *Profit Testing_2* in the shared Excel file online.

UNIT-LINKED CONTRACTS

For some contracts, other funds (e.g. unit fund for unit-linked assurances, reserves for traditional assurances) provide cashflows to the non-unit fund. In such cases these funds will need to be projected so that the ***expected cashflows*** to the non-unit fund can be determined. These calculations will require data items about the contract, e.g. proportion of premium allocated to purchase of units, bid-offer spread in unit prices; and assumptions which form a ***basis*** for the calculations, e.g. growth rate of unit fund, mortality and interest rate basis used to calculate ***required reserves***.
These expected cashflows, together with the direct expected cashflows into and out of the non-unit fund, are the components of the projected revenue account. The calculation of the direct expected cashflows will also require data items about the contract, e.g. initial and renewal expenses; and assumptions to form a basis, e.g. mortality of policyholders, rate of return earned on the non-unit fund. Unit-linked contracts require a unit reserve, which is equal to the unit fund value at any particular time, and a non-unit reserve. The calculation of the non-unit reserve follows the procedure described in the spreadsheet, which is referred to as ***zeroizing negative cashflows***.

Click here to see tab *UL Contracts* in the shared Excel file online.

Click here to see tab *UL Contracts (contd.)* in the shared Excel file online.

Click here to see tab *UL Contract Non-Unit Reserve* in the shared Excel file online.